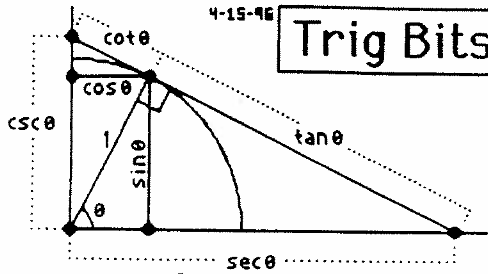
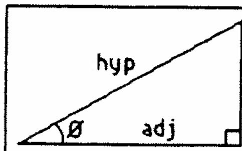
Rate·time + Rate·time = 1   Dist = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Midpoint $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

4-15-96

Trig Bits



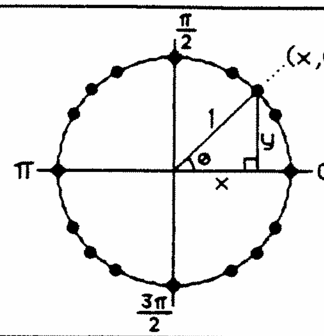


Sin Ø = $\frac{\text{opp}}{\text{hyp}}$ Tan Ø = $\frac{\text{opp}}{\text{adj}}$

Cos Ø = $\frac{\text{adj}}{\text{hyp}}$ opp = hyp·sin Ø
adj = hyp·cos Ø

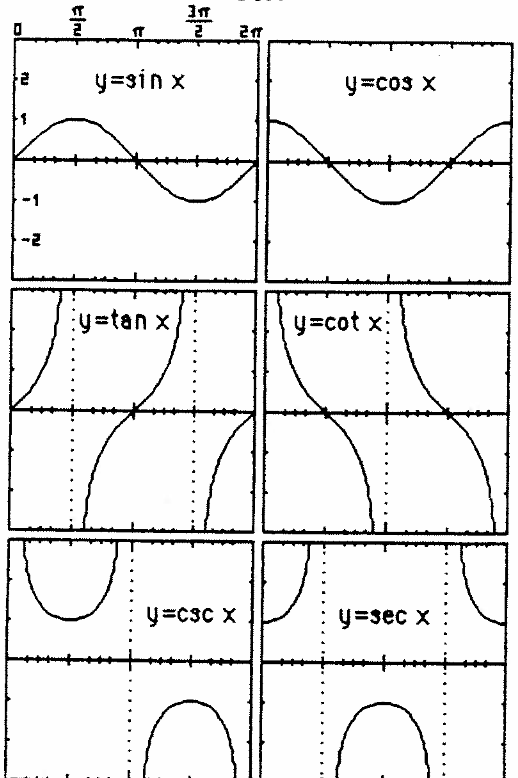
On the Unit Circle

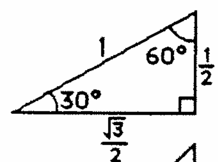
Sinθ = y
Cosθ = x
Tanθ = $\frac{y}{x} = \frac{\text{Sin}\theta}{\text{Cos}\theta}$
Cotθ = $\frac{x}{y} = \frac{\text{Cos}\theta}{\text{Sin}\theta}$
Secθ = $\frac{1}{x} = \frac{1}{\text{Cos}\theta}$
Cscθ = $\frac{1}{y} = \frac{1}{\text{Sin}\theta}$

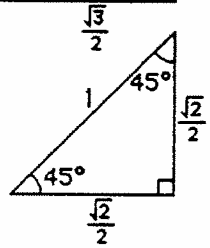


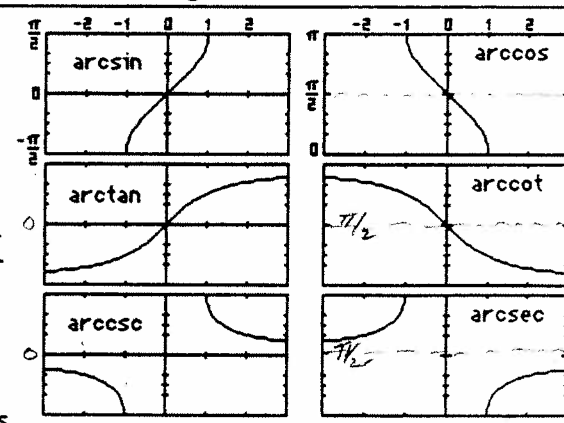
Positive (A,S,T,C)

SinØ CscØ	All
TanØ CotØ	CosØ SecØ









Addition Identities

Sin(a ± b) = Sin a Cos b ± Cos a Sin b
Cos(a ± b) = Cos a Cos b ∓ Sin a Sin b
Tan(a ± b) = $\frac{\text{Tan a} \pm \text{Tan b}}{1 \mp \text{Tan a Tan b}}$

Negative Angles

Sin(−a) = −Sin(a)
Cos(−a) = Cos(a)
Tan(−a) = −Tan(a)

m°	m^r	SinØ	CosØ	TanØ	CotØ	CscØ	SecØ	Arc Ansr
0	0	0	1	0	Undef	Undef	1	0
30	π/6	1/2	√3/2	√3/3	√3	2	2√3/3	π/6
45	π/4	√2/2	√2/2	1	1	√2	√2	π/4
60	π/3	√3/2	1/2	√3	√3/3	2√3/3	2	π/3
90	π/2	1	0	Undef	0	1	Undef	π/2
120	2π/3	√3/2	−1/2	−√3	−√3/3	2√3/3	−2	2π/3
135	3π/4	√2/2	−√2/2	−1	−1	√2	−√2	3π/4
150	5π/6	1/2	−√3/2	−√3/3	−√3	2	−2√3/3	5π/6
180	π	0	−1	0	Undef	Undef	−1	π
210	7π/6	−1/2	−√3/2	√3/3	√3	−2	−2√3/3	
225	5π/4	−√2/2	−√2/2	1	1	−√2	−√2	
240	4π/3	−√3/2	−1/2	√3	√3/3	−2√3/3	−2	
270	3π/2	−1	0	Undef	0	−1	Undef	−π/2
300	5π/3	−√3/2	1/2	−√3	−√3/3	−2√3/3	2	−π/3
315	7π/4	−√2/2	√2/2	−1	−1	−√2	√2	−π/4
330	11π/6	−1/2	√3/2	−√3/3	−√3	−2	2√3/3	−π/6
360	2π	0	1	0	Undef	Undef	1	0

Half Angle Formulas

$\text{Sin}\frac{\emptyset}{2} = \pm\sqrt{\frac{1-\text{Cos}\,\emptyset}{2}}$

$\text{Cos}\frac{\emptyset}{2} = \pm\sqrt{\frac{1+\text{Cos}\,\emptyset}{2}}$

$\text{Tan}\frac{\emptyset}{2} = \frac{1-\text{Cos}\,\emptyset}{\text{Sin}\,\emptyset} = \frac{\text{Sin}\,\emptyset}{1+\text{Cos}\,\emptyset}$

Double Angle Formulas

Sin(2Ø) = 2 Sin Ø Cos Ø
Cos(2Ø) = $\text{Cos}^2\emptyset - \text{Sin}^2\emptyset$
Cos(2Ø) = $2\,\text{Cos}^2\emptyset - 1$
Cos(2Ø) = $1 - 2\,\text{Sin}^2\emptyset$

Triangle Identities

$\text{Sin}^2\emptyset + \text{Cos}^2\emptyset = 1$
$1 + \text{Tan}^2\emptyset = \text{Sec}^2\emptyset$
$1 + \text{Cot}^2\emptyset = \text{Csc}^2\emptyset$

Cofunction Identities

$\text{Sin}\left(\frac{\pi}{2} - \emptyset\right) = \text{Cos}\,\emptyset$
$\text{Cos}\left(\frac{\pi}{2} - \emptyset\right) = \text{Sin}\,\emptyset$
$\text{Tan}\left(\frac{\pi}{2} - \emptyset\right) = \text{Cot}\,\emptyset$

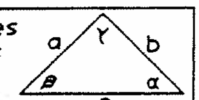
Conversions: $m^\circ \cdot \frac{\pi}{180^\circ} = m^r$ $m^r \cdot \frac{180^\circ}{\pi} = m^\circ$

Aproximate π's: 22/7 or 355/113

y = A Sin[B(x − C)] + D
y = A Cos[B(x − C)] + D

A = Amplitude
B = Frequency in 2π
(2π/B = Period)
C = x shift
D = y shift

Laws of Sines and Cosines



$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$

$a^2 = b^2 + c^2 - 2\cdot b\cdot c\cdot \text{Cos}\,\alpha$

Conics $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

Circle w/ B = 0 and A = C

Ellipse & Circle $B^2 - 4AC < 0$
Parabola $B^2 - 4AC = 0$
hyperbola $B^2 - 4AC > 0$

$y = \frac{1}{4c}x^2$ opens up/down

$x = \frac{1}{4c}y^2$ opens right/left

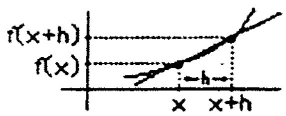
c = dist from vertex to focus

$S_sA \Rightarrow 1\Delta$ A acute $a \geq b$

$sSA = \emptyset$ Sin^{-1} Not ~~Possible~~   A obtuse $a < b$

$SA_s \Rightarrow$ Sin^{-1} Possible   A acute $b\,\text{Sin}A < a < 1$

Calculus



$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

Exponential Growth and Decay

$$y = Ce^{kt}$$

Integrate by parts

$$\int u\,dv = uv - \int v\,du$$

Vertical Motion in Feet and Seconds

$a(t) = v'(t) = s''(t)$

$s(t) = -16t^2 + V_0 t + S_0$

$v(t) = -32t + V_0$

$a(t) = -32$

Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

$$\text{Area} = \int_a^b f(x)\,dx$$

Volumes of Rotation

Discs $\quad \pi\int r^2\,dx$

Washers $\quad \pi\int R^2 - r^2\,dx$

Shells $\quad 2\pi\int rh\,dx$

Newton's Method

Where r is an approximation of a root of a function f(x):

$$\text{next approximation} = r - \frac{f(r)}{f'(r)}$$

L'Hopital's Rule

If $f(x)/g(x)$ has the indeterminate form of $0/0$ or ∞/∞ at the point c then:

$$\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$$

Other Tid Bits

$\frac{c}{0}$ indicates Vert. Asymptote

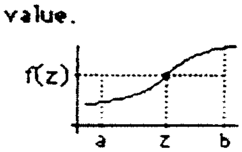
$\lim_{x\to\infty} f(x) = c$ Horiz. Asymptote

$\frac{0}{0}$ indicates hole in the graph

Average Value From a to b on a continuous function there is a z such that:

- At z, f(x) takes on the average value.
- f(z) is the average value.

$$f(z) = \frac{\int_a^b f(x)\,dx}{b-a}$$



$\frac{d}{dx}(ax^n) = nax^{n-1}$

$\frac{d}{dx}(uv) = u'v + uv'$

$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - uv'}{v^2}$

$\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(\text{Ln } x) = \frac{1}{x}$

$\frac{d}{dx}(\text{Sin } x) = \text{Cos } x$

$\frac{d}{dx}(\text{Cos } x) = -\text{Sin } x$

$\frac{d}{dx}(\text{Tan } x) = \text{Sec}^2 x$

$\frac{d}{dx}(\text{Cot } x) = -\text{Csc}^2 x$

$\frac{d}{dx}(\text{Sec } x) = \text{Sec } x \text{ Tan } x$

$\frac{d}{dx}(\text{Csc } x) = -\text{Csc } x \text{ Cot } x$

$\frac{d}{dx}(\text{arcSin } x) = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\text{arcCos } x) = \frac{-1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\text{arcTan } x) = \frac{1}{1+x^2}$

$\frac{d}{dx}(\text{arcCot } x) = \frac{-1}{1+x^2}$

$\frac{d}{dx}(\text{arcSec } x) = \frac{1}{x\sqrt{x^2-1}}$

$\frac{d}{dx}(\text{arcCsc } x) = \frac{-1}{x\sqrt{x^2-1}}$

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$

$\int e^x\,dx = e^x + c$

$\int \frac{1}{x}\,dx = \text{Ln}|x| + c$

$\int \text{Sin } x\,dx = -\text{Cos } x + c$

$\int \text{Cos } x\,dx = \text{Sin } x + c$

$\int \text{Tan } x\,dx = -\text{Ln}|\text{Cos } x| + c$

$\int \text{Cot } x\,dx = \text{Ln}|\text{Sin } x| + c$

$\int \text{Sec } x\,dx = -\text{Ln}|\text{Sec } x - \text{Tan } x| + c$

$\int \text{Csc } x\,dx = \text{Ln}|\text{Csc } x - \text{Cot } x| + c$

$\int \text{Sec}^2 x\,dx = \text{Tan } x + c$

$\int \text{Csc}^2 x\,dx = -\text{Cot } x + c$

$\int \text{Sec } x \text{ Tan } x\,dx = \text{Sec } x + c$

$\int \text{Csc } x \text{ Cot } x\,dx = -\text{Csc } x + c$

$\int \frac{1}{\sqrt{1-x^2}}\,dx = \text{arcSin } x + c$

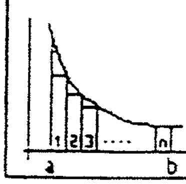
$\int \frac{1}{1+x^2}\,dx = \text{arcTan } x + c$

$\int \frac{1}{x\sqrt{x^2-1}}\,dx = \text{arcSec } x + c$

y' -- Slope $\quad$ + $\quad$ 0 $\quad$ − $\qquad$ y' = 0 or Ø → possible Max or Min

y'' -- Concavity $\quad$ + $\quad$ 0 $\quad$ − $\qquad$ y'' = 0 or Ø → possible Pt of Inflection

Approximate area using right handed rectangles



Width of each rectangle is:

$$w = \frac{b-a}{n}$$

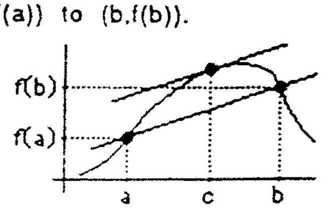
Area under f from a to b is:

$$\text{Area} = \lim_{n\to\infty}\left(w\sum_{i=1}^{n} f(a+wi)\right)$$

Trapezoidal Rule (n is the number of trapezoids)

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2n}\left(f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + f(x_n)\right)$$

if a funciton is continuous and differentiable from a to b, then there is a c so that the slope at c is the same as the slope from (a,f(a)) to (b,f(b)).



The Mean Value Theorem

$$f'(c) = \frac{f(b) - f(a)}{b-a}$$

Integration by Parts, Log, Inverse, Algebraic, Trig, Exponential

Reassociation

original problem

integral evaluated @ upper limit times the derivative of upper limit

$\int_a^{f(x)} f(t)\,dt$

$\sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right)$

$f^{-1}(x) = g(x)$

$g'(x) = \frac{1}{f'(g(x))}$

Principle Branches

$\sin^{-1}\ [-\pi/2, \pi/2]$

$\cos^{-1}\ [0, \pi]$

$\tan^{-1}\ (-\pi/2, \pi/2)$

$\cot^{-1}\ (0, \pi)$

$\sec^{-1}\ [0, \pi],\ x \neq \pi/2$

$\csc^{-1}\ [-\pi/2, \pi/2],\ x \neq 0$